AN EXTENDED FIELD BALANCING PROCEDURE FOR FLEXIBLE ROTORS FULLY LEVITATED BY ACTIVE MAGNETIC BEARINGS

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Abstract
An extended influence coefficient method (EICM) is presented in the present work to identify inherent unbalances in a flexible rotor levitated on active magnetic bearings (AMBs). With the help of trial sets of unbalances the traditional influence coefficient flexible rotor balancing procedure uses measured unbalance responses to obtain residual unbalances at predefined balancing planes. Due to partial attenuation of unbalance responses by active magnetic bearings in rotors mounted on active magnetic bearings, difficulty arises in estimating these influence coefficients since responses do not reflect real effects of trial unbalances. The EICM is presented to correlate magnetic forces and applied unbalance forces in the form of trial unbalances. This matrix is identified initially and helps in finding residual unbalances in the rotor system. The finite element method is used to model a flexible five-disc rotor system mounted on two active magnetic bearings for the numerical simulation of an experimental test rig. AMBs uses PID controller. The proposed procedure is applicable for both, discrete (constant) speeds and for run up/down measurements of magnetic forces. The developed procedure is benchmarked experimentally.

Keywords: Balancing; Flexible rotors; Residual unbalances; Active magnetic bearings;

1. Introduction
Residual unbalances beyond certain level in rotors are undesirable as it generates excessive unbalance forces and due to this unwanted large vibration takes place. At increasing rotor speed, a small amount of unbalance can prove to be fatal, which may cause the failure of the entire rotor system. Rotors manufactured by casting, machining, etc. processes cannot give a perfectly balanced condition; and hence, they must be balanced after manufacturing. A balanced rotor during operation, due to many reasons (like abrasion, acquiring lumped mass in the form of dust, etc.), may lose its balance state. So an operational rotor demands inspection and balancing at regular interval. Due to these reasons, the field balancing of rotors play an especially important role. Over time various balancing methods were defined and developed [1-6]. At present, the problem of balancing of rotor is broadly classified into two major divisions, viz. the off-line balancing method and the online automatic balancing method. Moreover, the rotor itself can be classified as rigid rotor or as flexible rotor. As a rule of thumb, if a rotor rotates below its first critical speed, the rotor can be considered as rigid rotor. At higher speeds, the rotor becomes flexible and the unbalance forces tend to bend the rotor. The present trend is to use active devices e.g. active magnetic bearings, for high-speed flexible rotors [7].

Now with requirement of very higher speeds, rotating machineries demand the active control through magnetic bearings so as to have contactless motion of rotors to reduce friction and eliminate the
lubrication system. AMBs suppress both steady-state unbalance responses due to residual unbalances and transient responses due to unexpected disturbances. For rotors mounted with AMBs, it is always economical if residual unbalances of rotor system could be reduced to the extent possible especially using infield balancing techniques to save time and practical difficulty of dismantling and re-commissioning of the rotor after the balancing. This also helps in reducing efforts of the controller and reduction in power consumption by the actuator in the form of continuous current to be supplied to attenuate steady-state responses due to residual unbalance forces. However, for rotors mounted on AMBs, the conventional flexible rotor balancing procedures fail to work since unbalance responses do not reflect actual effect of unbalances due to the active attenuation of rotor responses.

There is a wealth of literature available related with the control of rotors by active (including magnetic) bearings. One of the earliest research on active balancing was carried out by Van De Vegte [8]. He employed an active mass redistribution balancer system. The only input to the active balancing system was the measurement of the imbalance-induced vibration at support bearings. Zhou and Shi [7] made a survey on the active balancing and the vibration control of rotating machineries. They categorized active vibration control techniques into two majors, namely the direct active vibration control and the active balancing techniques. It was concluded that active balancing can suppress the imbalance induced vibration as they eliminate the root cause of vibration, the inherent imbalance of the rotor. Liu (2005) described a balancing technique for flexible rotor based on holo-spectrum. A balancing method known as low-speed holo-balancing was developed, which could balance a flexible rotor without test runs at high speeds. Lee et al. [9] developed an active balancing program using influence coefficient method and active balancing device of an electro-magnetic type and applied it to a high speed spindle system. Experiments were performed upon consecutive balancing with the operation speed changing without stopping spindle. Kang et al. [10] optimized the balancing of flexible rotor by minimizing the condition number of the influence coefficient matrix. This was achieved by varying the position of the sensors and balancing planes over the rotor shaft.

Ehyaei and Moghaddam [11] presented analytical and numerical investigations of a system of unbalanced flexible rotating shaft equipped with n-automatic ball balancers, where the unbalance masses were distributed in the length of the shaft. They concluded that to obtain the partial balancing the rotating speed should be more than the first natural frequency, closer distance between auto ball balancer and unbalance masses suppress vibration better. Pennachi et al. [12] investigated the rotor balancing problem using the influence coefficient method along with weighted least-square method. They concluded that the use of robust estimator allows successful determination of unbalance and automatic selection of weights without any expert’s knowledge. Sudhakar and Sekhar [13] investigated a model based method for fault identification in rotating systems. They identified the unbalance of rotor using the vibration minimization method. Rodrigues et al. [14] presented a non-linear bifurcation analysis of two-plane automatic balancing device for rotating machinery. From the symmetry properties they demonstrated that operation above both critical frequencies was a necessary but not sufficient condition for the stability of the balanced state.

In the present work an extended influence coefficient method for flexible rotor balancing is proposed. The technique is suitable for modern rotors that are fully mounted on AMBs. It is based on measured magnetic forces and unbalance responses at the AMB locations. A numerical example illustrates the proposed balancing method in which numerically generated unbalance responses and the AMB current information are used to estimate the unbalance forces. The numerical rotor model which is modelled using finite beam elements contains a five-disc flexible rotor system and levitated by two AMBs. The estimates of assumed residual unbalances are found to be excellent even with the addition of measurement noise in responses and currents.

2. Identification Procedure for Residual Unbalances

The conventional influence coefficient method considers a set of balancing planes and another set of measuring planes as shown in Fig. 1. In balancing planes alternatively a set of trial masses are kept
and its effect are measured at measuring planes in the form of unbalance responses. For rotors mounted on AMBs (Fig. 2) there are sets of balancing planes as for the conventional method, however, measuring planes are at AMB locations itself. Here, measurements are taken of the unbalance displacement responses and of the current supplied to AMBs. Both types of measurements are used for the flexible rotor infield balancing of residual unbalances.

Figure 1 The conventional Influence coefficient method (A rotor system with conventional bearings, balancing planes and measuring planes)

Figure 2 The modified influence coefficient method (A rotor system with active magnetic locations and balancing planes)

In general, the rotor can have discrete unbalances at disc locations, distributed unbalances along the shaft, or a combination of the both. In the present method, a number of balancing planes along the shaft are used (Fig. 2) to find equivalent residual unbalances in these balancing planes. AMBs are used to levitate the rotor on air and to attenuate rotor responses. These dynamic responses could result from the steady-state residual unbalances as well as transient disturbances in the rotor system. The relationships between trial masses (inputs) and corresponding magnetic forces (outputs) are developed by the extended influence coefficient methods.

Let us choose $p$ as the number of balancing planes (where the correction mass can be added or chipped-off), and $q$ as the number of measuring planes, i.e. at AMB locations. It should be noted that the aim of the present work is to estimate the equivalent residual unbalances at balancing planes and not at AMB locations so that after a successful estimation, the unbalance distribution is changed at these planes only. Accordingly, the number of balancing planes could be chosen depending upon the operational speed of interest of the rotor [1, 5]. First, the distributed residual unbalance $e(z)$ and the discrete residual unbalances are replaced by discrete equivalent unbalances acting within the chosen planes. Let the equivalent residual unbalances, $(U_i = m_i e_i)$, in each of the balancing planes be $\hat{U}_1, \hat{U}_2, \ldots, \hat{U}_p$. Then magnetic forces can be related to residual unbalances through a modified influence coefficients which is defined as

$$
\left\{ \hat{\mathbf{f}}_m \right\} = \left[ \hat{\alpha}_{q_1} \quad \hat{\alpha}_{q_2} \quad \cdots \quad \hat{\alpha}_{q_p} \right] \left\{ \hat{\mathbf{U}} \right\}
$$

or

$$
\left\{ f_m(\omega) \right\}_{q,p} = [\alpha(\omega)]_{q,p} \left\{ U \right\}_{q,p}
$$

Herein, $f_m$ is the measured magnetic forces at the AMB locations, $[\alpha(\omega)]$ is the modified influence coefficient matrix that relates the magnetic forces with the residual unbalances at balancing planes, $\omega$ is the rotor speed; and subscripts in matrices and vectors represent their sizes. Measurements are taken
at several speeds (at discrete few speeds or for several speeds corresponding to run-up and run-down characteristics of the rotor). Rewriting equation (1) for each of the speeds yields

\[
\{f_m\}_{qosl} = [\alpha]_{qosl} \{U\}_{pold} \quad \text{with} \quad \{f_m\} = \begin{bmatrix} f_m(\omega_1) \\ \vdots \\ f_m(\omega_n) \end{bmatrix} ; \quad [\alpha] = \begin{bmatrix} \alpha(\omega_1) \\ \vdots \\ \alpha(\omega_n) \end{bmatrix} ; \quad [U] = \begin{bmatrix} \tilde{U}_1 \\ \vdots \\ \tilde{U}_p \end{bmatrix}, \tag{2}
\]

where the subscript \(n\) in \(\omega\) represents the number of speeds at which measurements are taken. Once the modified influence coefficient matrix \([\alpha]\) is known for all speeds of interest, the unbalance distribution is determined according to equation (2) as

\[
[U]_{pold} = ([\alpha]^T[\alpha])^{-1} [\alpha]^T_{pold} \{f_m\}_{qosl}. \tag{3}
\]

However, \([\alpha]\) is yet not known; and the procedure of obtaining it experimentally, is explained in the following.

The modified influence coefficient matrix can be estimated experimentally by attaching a trial mass in balancing planes successively, and measuring displacements and currents at AMB locations for each case. At each particular speed, \(\omega_i\) with \(i = 1, 2, \ldots, n\), we obtain

\[
\begin{bmatrix}
\tilde{f}_{m_1}(\omega) \\
\tilde{f}_{m_2}(\omega) \\
\vdots \\
\tilde{f}_{m_i}(\omega) \\
\vdots \\
\tilde{f}_{m_q}(\omega)
\end{bmatrix} = \begin{bmatrix}
\alpha_{11}(\omega) & \alpha_{12}(\omega) & \ldots & \alpha_{1p}(\omega) \\
\alpha_{21}(\omega) & \alpha_{22}(\omega) & \ldots & \alpha_{2p}(\omega) \\
\vdots & \vdots & \ddots & \vdots \\
\alpha_{q1}(\omega) & \alpha_{q2}(\omega) & \ldots & \alpha_{qp}(\omega)
\end{bmatrix} \begin{bmatrix}
\tilde{U}_1 + \tilde{T}_R \\
\tilde{U}_2 \\
\vdots \\
\tilde{U}_p
\end{bmatrix}, \tag{4}
\]

where the second subscript in \(f_m\) represents magnetic force measurements while keeping the trial mass at that plane and \(\tilde{T}_R\) represents the trial unbalance. It is assumed that the trial masses for each measurement are sufficiently small such that the modified influence coefficients do not change. On subtracting equation (4) from equation (1), we get

\[
\begin{bmatrix}
\tilde{\alpha}_{11}(\omega) \\
\tilde{\alpha}_{21}(\omega) \\
\vdots \\
\tilde{\alpha}_{q1}(\omega)
\end{bmatrix} = \begin{bmatrix}
\{\tilde{f}_{m_1}(\omega) - \tilde{f}_{m_i}(\omega)\}/\tilde{T}_R \\
\{\tilde{f}_{m_2}(\omega) - \tilde{f}_{m_i}(\omega)\}/\tilde{T}_R \\
\vdots \\
\{\tilde{f}_{m_q}(\omega) - \tilde{f}_{m_i}(\omega)\}/\tilde{T}_R
\end{bmatrix}, \quad \text{with} \quad i = 1, 2, \ldots, n. \tag{5}
\]

Similarly, by attaching a trial mass in plane 2, the second column of the modified influence coefficient matrix in equation (5) is obtained. The above analysis is done at a constant speed, \(\omega_i\). Similarly, the modified influence coefficient matrix for all other speeds of interest \((i = 1, 2, \ldots, n)\) can be determined.

3. The Numerical Model of the Rotor-AMB System

The proposed algorithm is tested for a rotor system mounted on AMBs as shown in Fig. 3 using direct numerical simulation. The shaft contains five discs, including the two at AMB locations. The shaft is driven by a motor through a flexible coupling. The mass of the coupling is negligible small. The rotor is modeled by finite beam elements methods including gyroscopic effects [15]. Table 1 provides various parameters of the rotor and Table 2 gives specification of AMBs. The PID controller has the
following properties: \(K_p = 4200 \text{ Amp/m}, K_I = 2000 \text{ Amp/m-s}, \) and \(K_D = 3.0 \text{ Amp-s/m}.\) The current and the displacement stiffness coefficients are given, respectively, as \(k_i = 42.1 \text{ N/Amp} \) and \(k_s = 1.0521 \times 10^5 \text{ N/m}.\) Through the present numerical model with SIMULINK the simulation has been done to obtain bearing forces with the help of simulated current supplied to actuators and displacements at AMBs.

![Figure 3 The rotor-AMB model for the numerical simulation](image)

Table 1 Physical properties of the rotor

<table>
<thead>
<tr>
<th>S.N.</th>
<th>Parameter</th>
<th>Numerical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Disc 1: (m) (mass) and (I_d) (diametral mass moment of inertia)</td>
<td>0.78 kg and (4.44 \times 10^{-4} \text{ kg-m}^2)</td>
</tr>
<tr>
<td>2</td>
<td>Discs 2 and 5: (m) and (I_d)</td>
<td>0.88 kg and (5.17 \times 10^{-4} \text{ kg-m}^2)</td>
</tr>
<tr>
<td>3</td>
<td>Discs 3 and 6: (m) and (I_d)</td>
<td>1.20 kg and (13.10 \times 10^{-4} \text{ kg-m}^2)</td>
</tr>
<tr>
<td>4</td>
<td>Young’s modulus, (E)</td>
<td>(2.06 \times 10^{11} \text{ N/m}^2)</td>
</tr>
<tr>
<td>5</td>
<td>Mass density, (\rho)</td>
<td>(7850 \text{ kg/m}^3)</td>
</tr>
<tr>
<td>6</td>
<td>Shaft diameter, (d), and total shaft length, (L)</td>
<td>8 mm and 670 mm</td>
</tr>
<tr>
<td>7</td>
<td>Distances between various neighbouring discs (from left to right)</td>
<td>80 mm, 119 mm, 110 mm, 120 mm and 80 mm.</td>
</tr>
</tbody>
</table>

Table 2 Physical properties of active magnetic bearings

<table>
<thead>
<tr>
<th>S.N.</th>
<th>Parameter</th>
<th>Numerical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Number of poles</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>Magnetic bearing constant</td>
<td>(3.64 \times 10^6 \text{ Vsm/A})</td>
</tr>
<tr>
<td>3</td>
<td>Current-force constant</td>
<td>42.1 \text{ N/A}</td>
</tr>
<tr>
<td>4</td>
<td>Bearing stiffness</td>
<td>(1.052 \times 10^2 \text{ N/m})</td>
</tr>
<tr>
<td>5</td>
<td>Nominal air gap</td>
<td>0.80 mm</td>
</tr>
</tbody>
</table>

4. Numerical Simulations of Proposed Balancing Procedure

The estimation of residual unbalances has been done through two types of simulated data: (i) at discrete constant rotor speeds and (ii) at constantly accelerated run-up characteristics of the rotor. For both the cases a reference signal (i.e. with respect to a reference rotation angle from which the trial unbalance angular position has been defined) is used to obtain the phase of disc displacements and AMB currents. Figure 4 shows a typical magnetic force variation over time for a run-up characteristic at a constant angular acceleration of 200 rad/s\(^2\) starting from the rotor at rest. Figure 5 shows the corresponding frequency distribution of the magnetic force in \(z\)-direction. The present method uses both, magnitude and phase information of the magnetic forces for the case of (i) residual unbalances only and (ii) residual unbalances along with the trial unbalances in the balancing planes (alternatively). Estimates of the residual unbalances were determined by evaluating equations (5) and (1) for two scenarios: at discrete rotor speed and during run-up. A comparison is listed in Table 3 and shows excellent agreement of the estimated residual unbalance, in magnitude as well as in phase orientation. In order to investigate the robustness of the method, random noise (1, 2 and 5 \%) was added to the displacement and current signals. Since unbalance is proportional to \(1 \times\) rotor frequency, hardly any effect is visible on the estimates on addition of the noise for the present case and estimated residuals are similar to that in Table 3.
Figure 4 A typical magnetic force variation of AMB 1 over time for a run-up characteristic with an angular acceleration of 200 rad/s²

Figure 5 Frequency distribution of the magnetic force in z-direction in Fig. 4

Table 3 The assumed and estimated residual unbalances

<table>
<thead>
<tr>
<th>S.N.</th>
<th>Disc location</th>
<th>Magnitude (kg-m)</th>
<th>Phase (degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Disc 1</td>
<td>0.2340×10⁻⁴</td>
<td>5.73</td>
</tr>
<tr>
<td>2</td>
<td>Disc 2</td>
<td>0.7200×10⁻⁴</td>
<td>68.75</td>
</tr>
<tr>
<td>3</td>
<td>Disc 3</td>
<td>0.3600×10⁻⁴</td>
<td>120.32</td>
</tr>
</tbody>
</table>
5. Experimental Investigation

![Rotor mounted on AMBs](image1)

The procedure for the experimental data collections are as follows:

1. Time of run-up and run-down characteristics: constant acceleration within 10s, 30s and 60s up to 5000 rpm (above fourth critical speed)
2. For each run four sets of measurements of displacements and currents at AMB locations corresponding to (i) initial residual unbalances (ii) trial unbalance at disc 1 (iii) trial unbalance at disc 3 and (iii) trial unbalance at disc 4
3. During each run, the rotation angle with respect to a reference was acquired in order to have estimates of the phase information of currents and displacements with respect to shaft reference position.

Typical displacement and current characteristics are shown in Figs. 7 and 8, respectively. Estimates of residual unbalances for different run-up and rundown rates are summarized in Tables 4 and 5. Fig. 9 shows the polar plot corresponding to the magnitude and phase of residual estimated for run-up case. From the polar plot it is evident that discs 1 and 3 have very little residual unbalance magnitudes whereas the disc 4 has appreciable amount of residual unbalance magnitude. Because of this reason for discs 1 and 2 the phase estimates are poor (see Fig. 9 (a & b)). From Fig. 9(c) it is evident that phase estimates are relatively better but due to noise effect in the signal it is scattered around some narrow reason.

![Displacement characteristics over time](image2)

Fig. 7 A typical displacement characteristics over time for a run-up
Fig. 8 A typical current characteristics over time during a run-up

Table 4 Estimates of residual unbalances from run-up measurement data up to 5000 rpm

<table>
<thead>
<tr>
<th>S.N.</th>
<th>Frequency range parameters</th>
<th>Assumed unbalance magnitude</th>
<th>Estimated unbalance magnitude</th>
<th>Assumed unbalance phase</th>
<th>Estimated unbalance phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10 s run-up 0-83.33 Hz</td>
<td>0.0029</td>
<td>0.0004</td>
<td>60.0</td>
<td>89.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0060</td>
<td>0.0027</td>
<td>-150.0</td>
<td>-166.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0036</td>
<td>0.0004</td>
<td>90.0</td>
<td>166.0</td>
</tr>
<tr>
<td>2</td>
<td>30 s run-up 0-83.33 Hz</td>
<td>0.0029</td>
<td>0.0001</td>
<td>60.0</td>
<td>-87.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0060</td>
<td>0.0037</td>
<td>-150.0</td>
<td>-170.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0036</td>
<td>0.0002</td>
<td>90.0</td>
<td>172.9</td>
</tr>
<tr>
<td>3</td>
<td>60 s run-up 0-83.33 Hz</td>
<td>0.0029</td>
<td>0.0006</td>
<td>60.0</td>
<td>-139.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0060</td>
<td>0.0025</td>
<td>-150.0</td>
<td>135.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0036</td>
<td>0.0002</td>
<td>90.0</td>
<td>71.9</td>
</tr>
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</table>

Table 5 Estimates of residual unbalances from run-down measurement data from 5000 rpm

<table>
<thead>
<tr>
<th>S.N.</th>
<th>Frequency range parameters</th>
<th>Assumed unbalance magnitude</th>
<th>Estimated unbalance magnitude</th>
<th>Assumed unbalance phase</th>
<th>Estimated unbalance phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10 s run-down 0-83.33 Hz</td>
<td>0.0029</td>
<td>0.0002</td>
<td>60.0</td>
<td>149.1</td>
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<td>-150.0</td>
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<tr>
<td></td>
<td></td>
<td>0.0036</td>
<td>0.0004</td>
<td>90.0</td>
<td>116.6</td>
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<tr>
<td>2</td>
<td>30 s run-down 0-83.33 Hz</td>
<td>0.0029</td>
<td>0.0010</td>
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<td>-172.8</td>
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<td>0.0060</td>
<td>0.0021</td>
<td>-150.0</td>
<td>-159.6</td>
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<td>0.0004</td>
<td>90.0</td>
<td>141.9</td>
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<td>3</td>
<td>60 s run-down 0-83.33 Hz</td>
<td>0.0029</td>
<td>0.0004</td>
<td>60.0</td>
<td>-108.5</td>
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<tr>
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<td></td>
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<td>0.0036</td>
<td>-150.0</td>
<td>161.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0036</td>
<td>0.0003</td>
<td>90.0</td>
<td>143.3</td>
</tr>
</tbody>
</table>
6. Conclusions
In the present work, the conventional influence coefficient method for the flexible rotor balancing was extended for the case where the rotor is levitated by active magnetic bearings. Magnetic forces are used in the developed algorithm instead of displacements only. The proposed procedure was illustrated by direct numerical simulations and found to be robust against measurement noise. An experimental validation of the procedure was performed but requires improvement in terms of averaging power spectral densities in order to cope with the noise level inherent in the control loop of the active bearing. The procedure developed has potential for balancing of flexible rotors with other kinds of active bearings.

7. Acknowledgements
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8. References


