Identification of Foundations in Rotating Machinery Using Modal Parameters

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ABSTRACT

The vibration behaviour of a rotor bearing foundation system (RBFS) is affected by its foundation. This paper develops a parameter identification procedure using rotor and foundation motion measurements to identify the modal parameters of an equivalent foundation. The restriction in earlier work to foundations with a diagonal mass matrix is here removed. The procedure is applied to an RBFS comprising a rotor supported by two hydrodynamic bearings mounted on a flexibly supported rigid foundation block. Numerical experiments show that with input data truncated to two digits, reasonable foundation identification is possible. Thus, the procedure promises to be applicable in the field.

1 INTRODUCTION

The dynamic behaviour of rotating machinery is significantly affected by its foundation [1, 2]. Modelling the foundation of such machinery is of practical interest since the availability of a sufficiently accurate foundation model is an invaluable asset for efficient operation and balancing. There are two common procedures for incorporating the foundation into the analysis of the overall RBFS. The first uses appropriate experimental vibration measurements to identify equivalent foundation parameters; the other models the foundation by finite-elements, this latter approach being limited by difficulty of modelling [2]. This paper follows the former approach and is concerned with developing a procedure which is applicable to existing turbomachinery installations without requiring rotor removal.

Such identification procedures invariably require as input data a knowledge of the forces transmitted to the foundation via the bearing pedestals as well as of the motion of the foundation at an appropriate number of locations. Provided the dynamic properties of the rotor are known (not regarded to be significant problem), such force information can be obtained from performance monitoring instrumentation already present on existing turbomachinery installations, viz. displacement transducers measuring the relative motion between the rotor journals and the bearing housings [3, 4], and accelerometers measuring the absolute motion of the foundation at the housings. Such a rotor-model-based force determination approach, which relies on knowledge of the rotor unbalance and on the dynamic properties of the rotor, has been experimentally proven to give satisfactory identification of the dynamic stiffness properties of a simple flexible pedestal bearing support in a laboratory test rig [4]. Thus, it is assumed for the foundation identification procedure to be developed below, that all externally applied forces to the foundation at the housing supports (due primarily to rotor unbalance) are available as input data, together with foundation motion measurements. This is so even if the actual rotor unbalance is unknown; whereupon all measurements need to be repeated with an added known unbalance [4].

Required is the determination of parameters which enable the actual foundation in the RBFS to be replaced by an 'equivalent foundation' which reproduces the vibration behaviour of the actual RBFS over the speed range of interest. One approach is to describe the foundation in terms of appropriate symmetric mass, damping and stiffness matrices, i.e. to directly identify the elements of these matrices. This was the approach adopted in refs [4] and [5]. The simple two degrees of freedom (DOF) foundation alluded to above [4] required the identification of nine parameters. In more general foundations, where there is coupling between the bearing supports, one needs to allocate more DOF to the equivalent foundation. Thus, for a rotor supported by two hydrodynamic bearing pedestals which are rigidly fixed to a flexibly supported rigid block, one would need at least six foundation DOF with sixty three parameters to identify an equivalent foundation. Such an RBFS was
investigated in ref. [5] where, for the equivalent foundation to accurately predict the unbalance response, one required seven equivalent foundations, each valid over a restricted part of the overall speed range. Hence, such a direct identification of matrix elements does not appear to be practical.

A more promising approach appears to be to identify appropriate modal parameters of the foundation. Assuming the number of foundation natural frequencies that are likely to be relevant, such approaches concentrate on ‘decoupled equations’, turn by turn, thereby minimising the number of parameters to be solved at any one time. Such a procedure was applied to the abovementioned six DOF foundation and excellent identification was achieved [6]. However, the procedure proved problematic as it was necessary to solve sets of nonlinear equations, necessitating iteration with attendant convergence problems. Also, the foundation equations of motion needed to be written with a diagonal mass matrix. A modification of this approach reduced the number of unknown parameters and utilised a more efficient iteration procedure, but still assumed a diagonal mass matrix [7].

The assumption of a diagonal mass matrix is too restrictive. For foundations with flexural modes, it is too approximate; and even when only rigid body modes are relevant, formulating the equations of motion to ensure a diagonal mass matrix can be difficult. Hence, this paper enhances the earlier modal parameter foundation identification procedures [6, 7] to allow for full symmetric mass matrices, thereby extending applicability to foundations with flexural modes. The procedure is numerically evaluated by applying it to the six DOF flexibly supported rigid foundation block of ref. [6]. Of major interest is the performance of the identified foundation when all input data (force and displacement measurements) are truncated to two digit accuracy to simulate achievable measurement accuracy in practical implementation.

2 NOTATION

\( A \) modified foundation modal matrix with elements \( a_{ij} \)

\( C \) foundation damping matrix

\( c \) modal damping matrix

\( D \) displacement transformation matrix [6]

\( f, F \) vector of forces acting on foundation, vector of complex amplitudes thereof

\( G \) force transformation matrix [6]

\( K, K_c \) foundation stiffness matrices

\( k \) modal stiffness matrix

\( M, M_c \) foundation mass matrices

\( m \) diagonal modal mass matrix with diagonal elements \( m_k \)

\( n \) number of foundation degrees of freedom

\( Q \) vector of complex amplitudes of modal displacements of the foundation

\( u, U \) vector of displacements in mass centre coordinates, vector of complex amplitudes thereof

\( x, X \) vector of displacements in translational coordinates, vector of complex amplitudes thereof

\( x, y, z \) coordinate axes at foundation mass centre

\( \Phi \) foundation modal matrix with elements \( \phi_{ij} \)

\( \lambda \) diagonal matrix of foundation eigenvalues with diagonal elements \( \lambda_k \)

\( \Omega \) excitation frequency, rotor speed

\( \omega_k \) \( k^{th} \) natural frequency of foundation
3 THEORY

For an RBFS such as schematically shown in Figure 1, the system excitation is due to rotor unbalance and hence is synchronous with excitation frequency $\Omega$. Assuming proportional damping, the equations of motion of a general $n$ DOF foundation may be written as:

$$M\ddot{x} + C\dot{x} + Kx = f,$$

where $M$, $C$ and $K$ are $n$ by $n$ symmetric matrices. The elements of the vector $x$ are the $n$ independent displacements chosen to coincide with convenient measurement locations which include the excitation force application points. The elements of the vector $f$ are the excitation forces acting at selected locations (e.g. for the foundation in Figure 1 these would be the forces transmitted to the foundation at the bearing supports). Assuming that the system response is periodic with fundamental frequency $\Omega$, one can write [4]:

$$-\Omega^2 MX + i\Omega CX + KX = f$$

(2)

or:

$$[-\Omega^2 I + i\Omega M^{-1}C + M^{-1}K]X = M^{-1} f$$

(3)

In the identification procedure, the elements of $X$, viz. $X_1$, $X_2$, ..., $X_n$, are obtained from foundation motion measurements, whereas the elements of $F$, viz. $F_1$, $F_2$, ..., $F_n$, are calculated from a knowledge of the rotor model, the rotor unbalance and rotor motion measurements at the bearing stations [4]. Letting $X = \phi Q$, eqn (2) becomes:

$$[-\Omega^2 \phi + i\Omega \phi + k]Q = \bar{\phi} F$$

(4)

or, defining $A$ such that $A^T = \phi^{-1}$, eqn (4) becomes:

$$[-\Omega^2 I + i\Omega + \lambda]A^T X - m^{-1} \bar{\phi} F = 0$$

(5)

Noting that $m = \phi^T M \phi$, eqn (5) can also be written as:

$$[-\Omega^2 I + i\Omega + \lambda]A^T X - A^T M^{-1} F = 0$$

(6)

Either eqn (5) or eqn (6) can be used for identification purposes. Thus, letting $P = M^{-1} F$, eqn (6) yields the $n$ identification equations ($k = 1, ..., n$):

$$\left( -\Omega^2 + i\Omega \lambda_k + \lambda_k \right) \sum_{j=1}^{n} a_{jk} X_j - \sum_{j=1}^{n} a_{jk} P_j = 0$$

(7)
The parameters to be identified in the \( k \)th equation are: \( \xi_k = c_k/m_k \), \( \lambda_k = k_k/m_k = \omega_k^2 \), \( a_{jk} \) \((j = 1, \ldots, n)\) and at most \( n(n+1)/2 \) elements in \( \mathbf{M}^{-1} \). Also, because the modified mode shape elements are relative values, one can assign \( a_{kk} = 1 \). Hence, the number of unknown parameter values per identification equation is at most \( (n+1)(n+2)/2 \), being dependent on the number of zero elements in \( \mathbf{F} \); and the minimum number of speed data sets needed to solve the resulting simultaneous equations (obtained by substituting for \( \Omega \) and the corresponding \( X_j \) and \( F_j \) into eqn (6)) is less than or equal to \( (n+1)(n+2)/4 \). These simultaneous equations are nonlinear, so an effective iterative solution strategy is required. The solution strategy adopted is explained in Section 4 below.

Having found the unknown parameters in each of the \( n \) identification equations, one has, in effect, obtained an equivalent foundation, and there are various means for using this 'foundation' to obtain the system responses. Since eqn (3) suffices to determine the system response, in order to define the equivalent foundation, one could determine the elements of the matrices \( \mathbf{M}^{-1} \mathbf{K} \) and \( \mathbf{M}^{-1} \mathbf{C} \) and those elements of \( \mathbf{M}^{-1} \) which appear in the vector \( \mathbf{P} \). Thus, having identified all the elements in \( \mathbf{A} \), \( \mathbf{A} \) and \( \mathbf{A} \) (and hence \( \mathbf{A} \)), one can find \( \mathbf{M}^{-1} \mathbf{K} \) from:

\[
\mathbf{M}^{-1} \mathbf{K} = \mathbf{F} \mathbf{A} \mathbf{F}^T
\]

with a similar expression for \( \mathbf{M}^{-1} \mathbf{C} \). Those elements of \( \mathbf{M}^{-1} \), which could not be identified because of the zero elements in \( \mathbf{F} \), do not affect the system response. Hence, one can assign arbitrary nonzero values to them to enable \( \mathbf{M}^{-1} \) to be inverted to obtain an adequate mass matrix \( \mathbf{M}^* \). On multiplying eqn (3) from the left by \( \mathbf{M}^* \), one obtains an equation similar to eqn (2) which will generate the correct system response. The actual values of \( \mathbf{M} \), should they be desired, can be recovered by insisting that \( \mathbf{M}^* \mathbf{M}^{-1} \mathbf{K} \) be symmetric.

Alternatively, if eqn (5) is used for identification purposes, one obtains the \( n \) identification equations \((k = 1, \ldots, n)\):

\[
(-\Omega^2 + i\Omega \xi_k + \lambda_k) \sum_{j=1}^{n} a_{jk} X_j - \sum_{j=1}^{n} \phi_j F_j/m_k = 0
\]

The parameters to be identified in the \( k \)th equation now are: \( \xi_k = c_k/m_k \), \( \lambda_k = k_k/m_k = \omega_k^2 \), \( a_{jk} \) \((j = 1, \ldots, n)\) and \( m_k \). The \( \phi_j \) \((j = 1, \ldots, n)\) are a function of \( \mathbf{A} \) and hence are automatically identified once \( \mathbf{A} \) has been identified. Again, because the modified mode shape elements are relative values, one can assign \( a_{kk} = 1 \). Hence, the number of unknown parameter values per identification equation is \((n+2)\); and the minimum number of speed data sets needed to solve the resulting simultaneous equations (obtained by substituting for \( \Omega \) and the corresponding \( X_j \) and \( F_j \) into eqn (5)) is \((n+2)/2 \). These simultaneous equations are nonlinear, so again an effective iterative solution strategy is required. The solution strategy adopted is explained in Section 4 below.
Having found the unknown parameters in each of the \( n \) identification equations, one has, in effect, obtained an equivalent foundation, and there are various means for using this 'foundation' to obtain the system responses. Thus, having identified all the elements in \( \lambda, \xi, A \) (and hence \( \varphi \)) and \( m \), one can find \( M \) from:

\[
M = AmA^T
\]  

(10)

with similar expressions for \( K \) and \( C \). One could then use eqn (2) to generate the system response.

4 NUMERICAL EXPERIMENTS

The flexibly supported rigid undamped foundation block, previously identified in refs [6] and [7], was selected to illustrate the proposed identification procedure. This foundation was selected because it has exactly six DOF \((n = 6)\). Figure 2 shows a set of suitable measurement locations on the upper surface of the block. Using these six independent displacements as DOF is referred to as the translational formulation. It allows for the application of the external force \( f \) in the \( x_2 \) and \( x_5 \) directions at the connection point \( C_1 \) and in the \( x_3 \) and \( x_6 \) directions at the connection point \( C_2 \). Such a choice of DOF does not result in a diagonal mass matrix. To achieve this, one can locate rectangular coordinate axes at the mass centre of the block, aligned in the direction of its principal axes of inertia. A diagonal mass matrix results if one then selects for the DOF the displacements of the mass centre in the directions of these coordinate axes and the rotations of the block about these axes. Using these alternative six independent displacements as DOF is referred to as the mass centre formulation. In this formulation the equations of motion of the foundation are similar to eqn (1) except the mass, damping and stiffness matrices \( M_c, C_c \) and \( K_c \) have different elements. The vector of displacements is then given by:

\[
u = [v, y, z, \theta_x, \theta_y, \theta_z]^T
\]  

(11)

and the force vector becomes \( Df \). Additionally, \( \mathbf{u} = \mathbf{Gx}, \mathbf{G}^T = \mathbf{D}^{-1} \) and \( \mathbf{M} = \mathbf{G}^T\mathbf{M_cG} \), with similar expressions for \( C \) and \( K \) [6].

Figure 3 shows the stiffnesses supporting the foundation block and their locations. For a mass of 502.49 kg, one obtains the principal moments of inertia \( I_x = 5.2765 \text{ kg}\cdot\text{m}^2, I_y = 71.761 \text{ kg}\cdot\text{m}^2 \) and \( I_z = 68.595 \text{ kg}\cdot\text{m}^2 \). With respect to the mass centre, the connection points \( C_1 \) and \( C_2 \) are at \((-L/2, H/2, 0)\) and \((3L/8, H/2, 0)\) respectively. For this undamped foundation, one can evaluate \( M_c, K_c \) and \( D \) directly, as well as \( M \) and \( K \) [6].

![Fig.2. Measurement locations and directions (W = 317.5mm, H = 158.75mm, L = 1270mm).](image-url)
For the RBFS in Figure 1, with rotor and bearing details as given in ref. [5] and with the above foundation, the steady state system response was calculated over the frequency range of 300 to 1450 rad/s in steps of 50 rad/s. using unbalances of \( U_1 = U_2 = 10^4 \) kg.m, \( U_2 = U_4 = 10^5 \) kg.m and \( U_3 = 10^6 \) kg.m, using in-house software [5]. The input data ‘measurements’ needed for identification were then the complex response amplitudes \( X \), and the complex force amplitudes \( F \) at these speeds, resulting in 48 input data sets. Parameter identifications were carried out using both the mass centre and the translational formulations, assuming that the ‘measurements’ were truncated to either to 5 or 2 significant digits. The 5 digit input data served to evaluate the potential validity of the identification procedure and to define the achievable accuracy of the adopted computational procedure by minimising the effect of measurement errors. The 2 digit input data better reflected attainable field measurement accuracy. Solutions using both eqns (7) and (9) were attempted using input data corresponding to both mass centre and translational formulations.

The solution procedure for the unknown parameters in each of the six identification eqn (7) involved a nested iterative approach wherein for initially assumed values for all elements of \( \mathbf{M}^{-1} \), an assumed \( \lambda_k \) was assigned to all terms involving \( a_{jk} (j \neq k) \). This resulted in 48 simultaneous linear equations with 10 or 11 unknowns (one of which was \( \lambda_k \)), depending on whether the translational or mass centre formulation was used, which equations were solved using least squares regression [5]. This was repeated for assumed \( \lambda_k \) values spanning the assumed speed range of interest, i.e. from \( \lambda_k = 9 \times 10^4 \) (rad/s)\(^2 \) (corresponding to \( \omega_k = 300 \) rad/s) to \( \lambda_k = 2.25 \times 10^6 \) (rad/s)\(^2 \) (corresponding to \( \omega_k = 1500 \) rad/s), using a simple search procedure coupled with interval halving to find the correct \( \lambda_k \) to some specifiable precision. On completing this for all six equations, one obtained an updated \( \mathbf{M}^{-1} \) and the whole process was repeated until the solutions converged. The solution procedure for the unknown parameters in each of the six identification eqn (9) was similar except one needed initially assumed values for \( \mathbf{\Phi}^T \) rather than for \( \mathbf{M}^{-1} \). One now obtained 48 simultaneous linear equations with 7 unknowns (one of which was again \( \lambda_k \)). One could, as a variation, update the assumed \( \mathbf{\Phi}^T \) and \( \mathbf{M}^{-1} \) to try to speed up convergence. The converged solutions then yielded the equivalent foundations.

5 RESULTS AND DISCUSSION

The first numerical experiments were for the mass centre formulation with 5 digit input data accuracy as these were the most accurate measurements available and the actual mass matrix, being diagonal, provided a convenient yardstick for gauging the achievable accuracy using either identification eqn (7) or (9). Table 1 shows the actual parameter values of the foundation as obtained from an eigenvalue solution of the homogeneous form of eqn (1) with \( \mathbf{C} = \mathbf{0} \). These parameter values were identified very well when a diagonal mass matrix was assumed even with only 2 digit input data, as illustrated for the natural frequencies and inertias in Table 2 [7].
Using first identification eqn (7), one obtained surprisingly accurately all the natural frequencies and modified mode shapes in the first pass, but the identifiable $M_{-1}$ parameters were poor. The second pass did not indicate any convergence tendencies; and indeed, even when the correct $M_{-1}$ was assumed, the solutions for the identifiable $M_{-1}$ parameters deteriorated. The high condition number of the least squares coefficient matrix indicated that the simultaneous equations were so highly ill-conditioned that inaccurate solutions could (and apparently did) result.

<table>
<thead>
<tr>
<th>Mode</th>
<th>$\omega$ (rad/s)</th>
<th>a(k,1)</th>
<th>a(k,2)</th>
<th>a(k,3)</th>
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<td>1.000</td>
<td>0.525</td>
<td>0.090</td>
<td>-0.004</td>
<td>-0.033</td>
<td>0.032</td>
<td>502.49</td>
<td>386.37</td>
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**Table 1** Natural frequencies, modified mode shapes, inertias and modal masses. Mass centre formulation.

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**Table 2** Natural frequencies and inertias – Mass centre formulation, diagonal mass matrix. (a – actual; 5 – 5 digit data; 2 – 2 digit data) [7]

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**Table 3** Natural frequencies, inertias and modal masses – Mass centre formulation. (a – actual; 5 – 5 digit data; 2 – 2 digit data)
Fig. 4: Comparison of unbalance responses. Mass centre formulation. Full mass matrix.

Fig. 5: Comparison of unbalance responses. Mass centre formulation. Diagonal mass matrix.

Using next identification eqn (9), very good identification was achieved with 5 digit input data, proving that the approach is valid in principle. The only problem was the choice of the initially assumed A matrix to preclude any likelihood of a singular least squares coefficient matrix. It was decided to use the identified A matrix obtained from the first pass when using the identification eqn (7), as these were apparently close to the actual values. This procedure was then repeated using 2 digit input data, and the identified natural frequencies, inertias (the diagonal elements of the mass matrix) and modal masses are summarised in Table 3. While the identification results obtained using 5 digit input data are excellent, there is noticeable error in some of the identified parameters when using 2 digit input data.

The effect of these errors on the suitability of the now obtained equivalent foundation can be seen in Figure 4 where the predicted unbalance response amplitudes at the end, quarter way along and halfway along the rotor of the RBFS in Figure 1 are compared with the actual ones. While there is excellent agreement between the actual responses and those obtained using 5 digit input data, the agreement using 2 digit input data is at best approximate. Figure 5 shows the same responses when the equivalent foundations were obtained assuming a
diagonal mass matrix [7]. Here the agreement between the actual responses and those obtained using 2 digit input data is excellent. Clearly, there is scope for improving the solution procedure.

Table 4 summarises the identified natural frequencies, inertias (the diagonal elements of the mass matrix) and modal masses that were obtained when using translational coordinates and Figure 6 compares unbalance response amplitudes at the end, quarter way and halfway along the rotor of the RBFS in Figure 1. Again, the 5 digit input data identifications are excellent and prove the validity of the procedure, while the 2 digit input data identifications still show room for improvement. Here, the mass matrix is not diagonal, and previously attempted identifications assuming a diagonal mass matrix proved extremely poor [7].

<table>
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<th>Mode</th>
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<td>$\omega_1$ (rad/s)</td>
<td>765.0</td>
<td>626.7</td>
<td>543.4</td>
<td>1259.</td>
<td>823.6</td>
<td>1002.</td>
</tr>
<tr>
<td>$M_1$ (kg, kg.m$^2$)</td>
<td>502.5</td>
<td>211.9</td>
<td>331.6</td>
<td>335.0</td>
<td>150.4</td>
<td>222.2</td>
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<tr>
<td>$m_1$ (kg, kg.m$^2$)</td>
<td>386.4</td>
<td>3.291</td>
<td>22.13</td>
<td>286.3</td>
<td>54.29</td>
<td>0.07415</td>
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<td>$\omega_5$ (rad/s)</td>
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<td>626.8</td>
<td>543.5</td>
<td>1259.</td>
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<tr>
<td>$M_5$ (kg, kg.m$^2$)</td>
<td>502.4</td>
<td>211.9</td>
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<tr>
<td>$m_5$ (kg, kg.m$^2$)</td>
<td>386.3</td>
<td>3.291</td>
<td>22.13</td>
<td>286.3</td>
<td>54.29</td>
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<tr>
<td>$\omega_2$ (rad/s)</td>
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<td>547.9</td>
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<td>223.3</td>
<td>346.6</td>
<td>339.5</td>
<td>170.2</td>
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<td>3.710</td>
<td>17.47</td>
<td>285.3</td>
<td>74.12</td>
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</table>

Table 4 Natural frequencies, inertias and modal masses – Translational formulation. (a – actual; 5 – 5 digit data; 2 – 2 digit data)

Fig. 6: Comparison of unbalance responses. Translational formulation. Full mass matrix.
6 SUMMARY OF CONCLUSIONS

Earlier foundation identification techniques using modal parameters were restricted to foundations with a diagonal mass matrix whereas the proposed technique can accommodate foundations with full symmetric mass matrices. It is therefore capable of catering for flexural as well as rigid body modes. Also, it avoids the need for coordinate transformations.

The proposed technique is valid in principle insofar as the identified equivalent foundation can accurately reproduce the response of an unbalanced RBFS over the speed range of interest provided the input measurement data used for identification is accurate to 5 significant digits. However, the reproduction of such unbalance responses with 2 digit input data was not everywhere so accurate and needs to be improved.

Once the solution procedures are improved so as to achieve good identification with 2 digit input data, the proposed procedure promises to be applicable in the field as it can utilise directly measurements available from existing monitoring instrumentation.

7 REFERENCE LIST


