Observer design for rotating shafts excited by unbalances

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ABSTRACT

One of the major problems in controller design for active rotor systems excited by unbalances is the fact that the unbalances acting on the shaft are never known to full extent. If state feedback is applied, this problem reduces to accurate state estimation despite the presence of unbalances. Two observer types accounting for this problem, a disturbance observer and an unknown input observer are applied to a rotor system with piezoelectric active bearings. The two approaches are compared with respect to resulting control performance, accuracy of the estimated states and expense for implementation.

1 INTRODUCTION

State space control offers various advantages over output control, such as the possibility for eigenstructure assignment, decoupling control or optimal LQ control. A disadvantage of state space methods is that the states of the system have to be known at all times. Hence, besides controller design, it is necessary to implement an observer in order to estimate the system states. If sinusoidal disturbances act on the plant to be observed, ordinary observers fail to approximate the system states accurately. In case of a rotating shaft, the inevitable unbalance leads to such a sinusoidal excitation. The obvious approach, i.e. integration of the unbalance excitation into the observer by means of additional inputs is infeasible because of the lack of knowledge about the actual unbalance. Neither its distribution along the shaft, nor its amplitude or phase can be identified to full extent. Apart from other aspects to consider in the context of controller and observer synthesis, this article focuses on the task of implementing an observer for systems excited by sinusoidal disturbances in isolation.

A first approach for accurate state estimation in the presence of disturbances, the disturbance observer (DO), was introduced by Johnson [1], [2]. In DO theory, it is assumed that the signal types of the disturbances acting on the system can be described by differential equations. These are included in the observer matrices in order to account for the effect of the disturbances. As a result, states as well as disturbances are observed, leading to improved accuracy of state estimation. Furthermore the estimated disturbance can be used to implement feed forward control approaches in the presence of not measurable disturbances. Comprehensive
studies on DOs by Johnson can be found in [3] and a good summary of Johnson’s results is given in [4].

Another method to deal with disturbances, the unknown input observer (UIO) was introduced by Wantanabe and Himmelblau in [5]. Since then it was widely used in the field of robust fault diagnosis and isolation, see [6], [7] for example. The structure of UIOs is based on the general Luenberger observer, and its matrices are designed in such a way, that the observer estimates the states properly despite any type of disturbances acting on the plant via a specific distribution. In contrast to the original application of the UIO in robust fault diagnosis, i.e. estimation of output signals, it is used to estimate the system states in this paper.

2 MODELLING AND CONTROLLER DESIGN

The observer types considered in this article will be analyzed in a test rig application by means of simulations. The rig consists of a rotor with a disc, a passive bearing and an active double bearing, equipped with four piezoelectric actuators. The rotor replicates the low pressure shaft of a jet engine in its mechanical attributes and exhibits four resonance frequencies in its operating range. A schematic of the rig is shown in Figure 1, where also the positions of eight displacement sensors are highlighted.

Figure 1: Schematic of the test rig

A finite element model of the rotor is derived on the basis of Timoshenko beam theory, while bearings are modeled using discrete stiffness, inertia and piezoelectric elements. Gyroscopic moments are neglected for the sake of simplicity. Modal reduction is applied to the overall model consisting of rotor, bearings and actuators, resulting in 8 modal degrees of freedom (DOF) and 16 DOF in state space respectively. Damping is introduced by means of viscous damping and a damping ratio of 0.8% for all modes. The governing equations read:

\[
\frac{\text{d}}{\text{d}t}X = AX + Bu + Ed
\]

where \( X \in \mathbb{R}^n \) is the vector of system states, \( u \in \mathbb{R}^{nu} \) is the control input, i.e. the voltages applied to the actuators, \( d \in \mathbb{R}^{nd} \) is the disturbance, i.e. forces acting on the shaft and \( Y \in \mathbb{R}^y \) is the system output, i.e. the displacement sensor signals. \( A, B, C, D, E \) are system matrices with appropriate dimensions. The model (1) is controllable and observable.
In order to be able to compare control performance in the presence of the different observers, a simple pole placement regulator is designed. The eigenfrequencies within operating range are reduced in order to reduce unbalance excitation at resonance and the damping ratios of the first 4 modes are increased to 10%. The resulting state space controller is denoted by \( K \), and the respective control input is \( u = -Kx \).

3 THEORY

The state of the art of DOs as well as UIOs and an algorithm for estimation of the disturbance input matrix is presented in this section. The system under consideration is assumed to be controllable and observable in the following.

3.1 Unknown input observer

In UIO theory, disturbances are considered as unknown inputs, i.e. no knowledge about the signal type of the disturbance is assumed to be available. UIOs are based on the general form of a Luenberger observer (2):

\[
\dot{\hat{x}} = F \hat{x} + TBu + K_{\text{ui}}y \\
\dot{\hat{x}} = z + H y
\]

Where \( K_{\text{ui}} = K_1 + K_2 \) and \( \hat{x} \) is an estimate of \( x \). If

\[
(HC - I)E = 0 
\]

and \( T = I - HC, F = TA - K_1C, K_2 = FH \), then the estimation error, \( \tilde{x} = x - \hat{x} \), can be described by [8]:

\[
\dot{\tilde{x}} = F\tilde{x}
\]

Thus, if all eigenvalues of \( F \) lie in the left half plane, \( \tilde{x} \) will go to zero. The matrix \( K_1 \) is a free parameter, which can be designed e.g. by pole placement of the system \((TA, C)\) in order to achieve the desired dynamics of \( \tilde{x} \).

It can be shown [8], that the observer described above is an UIO of (1) iff:

\[
\text{rank}(CE) = \text{rank}(E) \text{ and}
\]

\[
(C, TA) \text{ is a detectable pair}
\]

The conditions (5) and (6) imply two restrictions for UIOs:

1. Because of the rank condition (5), the maximum number of disturbances considered cannot be larger than the number of independent measurements, i.e. \( n_{\text{dr}} \leq \text{rank}(C) \) if the disturbance input matrix \( E \) is of full column rank.
2. Condition (6) is dependent on the disturbance input matrix $E$, which cannot be influenced specifically. In some cases instable poles of $TA$ are not observable and thus a stable $F$ cannot be found.

### 3.2 Disturbance observer

In many technical applications some knowledge about the signal type of the disturbance is available and it is possible to derive a homogeneous disturbance model which is capable of reproducing the disturbance. This fact is taken advantage of in the DO approach [4].

The disturbance model in its general form reads:

$$
\begin{align*}
\dot{\hat{x}}_d &= A_d \hat{x}_d \\
\dot{\hat{d}}_d &= C_d \hat{x}_d
\end{align*}
$$

(7)

where $\hat{d}_d$ represents the disturbance. In the case of a shaft rotating at constant frequency $f = \Omega/2\pi$ and being excited by unbalances, the disturbance model is given by

$$
A_d = \begin{bmatrix} 0 & I \\ -\Omega^2 I & 0 \end{bmatrix}, C_d = \begin{bmatrix} I & 0 \end{bmatrix}, \hat{d}_d \in \mathbb{C}^{\text{nr}}
$$

(8)

Assuming that the disturbance is fully described by (7), it is integrated into the observer matrices by means of additional DOF (9):

$$
\begin{align*}
\dot{\hat{x}}_B &= \begin{bmatrix} A & EC_d \\ 0 & A_d \end{bmatrix} x_B + \begin{bmatrix} B \\ 0 \end{bmatrix} u = A_d x_B + \begin{bmatrix} B \\ 1 \end{bmatrix} u \\
\dot{\hat{y}} &= \begin{bmatrix} C & 0 \end{bmatrix} x_B = C_B \hat{x}_B
\end{align*}
$$

(9)

Where $x_B = [\hat{x}^T, \hat{x}_d^T]^T$ and $\hat{x}$ is the desired approximation of the states. For the sake of simplicity it is assumed that there are no feedthrough components present. The DO can easily be extended to systems possessing feedthrough components [4].

A Luenberger observer is applied to the augmented system (9), i.e. a matrix $L$ is designed via pole placement in such a way that the observer states converge to the original states:

$$
\dot{\hat{x}}_B = A_d x_B + \begin{bmatrix} B \\ 0 \end{bmatrix} u + L(y - \hat{y})
$$

(10)

Assuming that the actual disturbance $d$ associated with some disturbance states $x_d$ can be described by (7) and all system matrices are not subject to modeling inaccuracy, the observation error $\tilde{x}_B = x_B - \begin{bmatrix} \hat{x} \\ x_d \end{bmatrix}$ is the solution to (11):

$$
\dot{\tilde{x}}_B = (A^* - LC_B) \tilde{x}_B
$$

(11)

Thus, the observer states converge to the real states despite the presence of disturbances if $L$ is chosen in such a way that $(A^* - LC_B)$ is stable. From a theoretical point of view, it is not sufficient to check stability of the overall system consisting of plant, regulator and observer by inspection of the eigenvalues of $A^* - LC_B$ for $\Omega \in [\Omega_{\min}, \Omega_{\max}]$ if $\Omega \neq \text{const}$. Nonetheless it is clear from experience that the system
will be stable if its time variation is slow and its eigenvalues have negative real parts for all \( \Omega \), a result which is given on a theoretical basis in [9]. Sufficient conditions for the stability of linear time-varying systems can be found in [10].

Before generating a suitable matrix \( L \) to place the poles of the system to be observed (9), it has to be checked for observability. A sufficient condition for (9) to be observable based on the Hautus observability criterion [11] is given below:

If the poles of the disturbance model \( \pm i\Omega \) are not part of the sets of the poles and transmission zeros [12] of the system \( A, E, C \), i.e.

\[
\det(\pm i\Omega - A) \neq 0 \text{ and rank } \begin{bmatrix} \pm i\Omega - A \\ C \\ 0 \end{bmatrix} = n + n_{dr},
\]

then the system (9) is observable.

It follows from (12) that \( \text{rank}(C) \geq n_{dr} \), i.e. the maximum number of disturbances to be observed must be smaller than or equal to the number of independent measurements for the augmented system to be observable.

It shall be noted that for each disturbance considered in the DO, there are two additional states in the augmented system (9). Thus, the number of disturbances considered should be small in order to keep the computational effort low in real time applications.

### 3.3 Estimation of the disturbance input matrix

A great drawback of both of the above approaches for state estimation is the fact that the way in which disturbances excite the system has to be known. To overcome this problem, an approximate disturbance input matrix can be derived on the basis of measurements using de-convolution method [8], [13].

Measurements of a run up or run down of the rotor system without control are utilized to estimate \( E \). The procedure is based on discrete time theory. Thus, the continuous time model (1) has to be transformed into discrete time:

\[
x(k + 1) = A_{\text{disc}}x(k) + B_{\text{disc}}u(k) + E_{\text{disc}}d(k) \\
y(k) = Cx(k)
\]

(13)

The residual \( r(k) \) at discrete time \( k \), i.e. the difference between the measured data \( y_M(k) \) and the response of the model \( y(k) \) to some unknown disturbance \( d(k) \) and an unknown initial condition \( x(0) \) can be described by:

\[
r(k) = y_M(k) - y(k) = y_M(k) - CA_{\text{disc}}^k x(0) - \sum_{i=1}^{k} CA_{\text{disc}}^{i-1} E_{\text{disc}}d(k - i)
\]

(14)

For every \( k \), \( d_1(k - 1) = E_{\text{disc}}d(k - 1) \) is obtained by setting \( r(k) = 0 \) and solving for \( d_1(k - 1) \):

\[
C d_1(k - 1) = y_M(k) - CA_{\text{disc}}^k x(0) - \sum_{i=2}^{k} CA_{\text{disc}}^{i-1} d_1(k - i), k = 1 \ldots N
\]

(15)
where \( N \) is the number of samples in the measurement. Note that a unique solution to (15) exists iff \( \text{rank}(C) = n \) [8]. Hence the number of independent measurements should not be smaller than the system order. If this condition is not satisfied, \( d_1(k) \) can only be approximated. As a result, the accuracy of \( E \) may be insufficient for accurate state estimation. It shall be noted, that this high number of sensors is only required for the process of determining \( E \) and not for the implementation of observers.

In the case of \( n \) independent measurements \( Cx(0) = y(0) \) can be solved without approximation in order to include the effect of \( x(0) \) in (15). If a smaller number of independent measurements, this equation can only be solved in the least squares sense. Since for stable systems \( \lim_{t \to \infty} A_{\text{disc}}^t = 0 \), we can also set \( x(0) = 0 \) in (15) in this case without introducing too much error if a long measurement is considered.

A set of \( N \) vectors \( d_i(k) \) is calculated from (15) and arranged in a matrix \( M \):

\[
M = [d_1(1), d_1(2), \cdots, d_1(N)]
\]

(16)

Since \( d_i(k) = E_{\text{disc}}(k) \), all \( d_i(k) \) must lie in a subspace of \( E_{\text{disc}} \). \( E_{\text{disc}} = M \) satisfies this condition, but recall that for the existence of UIOs and DOs, the number of columns of \( E \) must be less or equal to the number of independent measurements if \( E \) is of full column rank. An approximate matrix \( \tilde{E}_{\text{disc}} \) possessing \( q \) columns is obtained on the basis of a singular value decomposition of \( M \) [8]:

\[
M = U \begin{bmatrix}
\text{diag}(\sigma_1, \sigma_2, \cdots, \sigma_p) & 0 \\
0 & 0
\end{bmatrix} V^T \Rightarrow \tilde{E}_{\text{disc}} = U \begin{bmatrix}
\text{diag}(\sigma_1, \sigma_2, \cdots, \sigma_q) & 0 \\
0 & 0
\end{bmatrix}
\]

(17)

The matrices \( U, V \) are left and right singular matrices and \( \sigma_1, \cdots, \sigma_p \) are the singular values with \( \sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_q \geq \cdots \geq \sigma_p \). The desired approximation of \( E \) is obtained by transforming \( \tilde{E}_{\text{disc}} \) into continuous time.

4 APPLICATION TO A ROTOR TEST RIG MODEL

The techniques discussed in section 3 are applied to the model introduced in section 2. To simulate a close to reality situation, the unknown unbalance distribution is represented by a randomly distributed unbalance that leads to approximately the same displacement amplitudes as those measured at the rig during a slow run up. To be able to focus on the effect of the disturbance, no model inaccuracies are considered.

An approximate disturbance input matrix \( E \) with 4 columns is generated using the method presented in section 3.3. The sensors shown in Figure 1 and 8 additional sensors are used in order to achieve a good approximation of \( E \). The additional sensors are chosen in such a way, that the condition number of the resulting output matrix \( C \) is minimized. An UIO is implemented using all 4 columns of \( E \), while just the two most significant columns are used to implement a DO in order to keep the observer order small.

For the sake of comparison, an ordinary Luenberger observer is implemented. The poles of all observers are placed in the same region in the complex plane.
In the DO design procedure, the parameter dependence of $A^*$ in (11) on $\Omega$ has to be considered in the design of $L$. In the case of the test rig, it is sufficient to derive a single matrix $L$ at some specific design point to achieve the desired properties in the whole frequency range considered. The tradeoff to be made between sufficiently fast dynamics and low noise amplification is more severe as in the case of a time invariant observer since the poles of the observer migrate in the complex plane as the rotational frequency changes. $L$ is designed using a pole placement routine which guarantees some robustness against uncertainty in the system matrices [14].

Investigations showed that conditions (5), (6), though being sufficient for an UIO to exist, are not strict enough for an UIO which is implementable in a real time application to exist. The poles of $TA$ may differ significantly from those of $A$ and may lie far away from the origin. If those poles are not observable, they cannot be moved closer to the origin, possibly leading to high noise amplification or prohibiting implementation of the observer at low sampling frequencies in real time applications. Even if those poles are observable, great differences in the pole locations of $TA$ cause low accuracy in the locations of the poles of $F$ and high gains of $K_{ui0}$, leading to high sensor noise amplification and ill-conditioned observer matrices. Thus, a practically implementable UIO exists iff conditions (5) and (6) hold true and the poles of $TA$ lie at “acceptable” locations.

In the case of the test rig, some poles are moved far into the left half plane by $T$ and implementation of an UIO is not possible. Since $T = I - HC$ and $H$ is obtained by solving (3), e.g. by pseudo inversion, the only way to overcome this problem is to alter $C$. To show the potential of the UIO, a configuration is generated by reallocation of the 8 sensors such that the above condition for an UIO to be practically implementable holds true.

Implementation of the DO is possible by using the sensors in measurement plane B and C, see Figure 1. Thus, just half the number of sensors is required for DO-implementation in the case of the rig.

### 4.1 Frequency domain analysis

In order to survey the performance of the observers at steady state, the unbalance response is inspected in the frequency domain. The rotational frequency is normalized to the first eigenfrequency of the uncontrolled system in the following plots.

Figure 2 shows the displacement amplitudes in measurement plane D (see Figure 1) in the vertical direction and the voltage applied to the vertical actuator in plane A for different systems, i.e. the system without control as well as four types of controlled systems: A system with known system states (perfect control), an ordinary observer, a DO and an UIO. It is observed, that the behavior of the overall system consisting of plant and ordinary observer is quite different from the desired behavior. Control performance is reduced and control effort is increased significantly. Both the DO and the UIO do not alter the overall system to a considerable extend. The desired transfer behavior can be achieved in the presence of the unknown unbalance excitation.
Figure 2: Unbalance response and control effort in frequency domain

Figure 3 shows the relative estimation error, averaged over all states as well as the mean relative difference between the states of the systems including the respective observers and the states of the system with perfect control (i.e. known states). Both observer types presented in this article outperform the ordinary observer in terms of estimation error and difference in system states. It is observed, that the disturbance observer alters the system less severely, which is important when using the observer for controller design. The UIO leads to a more accurate approximation of the states, which is advantageous in the context of fault detection and isolation.

Figure 3: Estimation error in frequency domain

4.2 Time domain analysis
For the ordinary observer and the UIO, the capability of reproducing the states is proven by frequency domain analysis and inspection of the observer poles. This is not the case for the DO. The disturbance is assumed to be a solution to system (7), (8), which is able to reproduce harmonic disturbances. During a run up, the disturbance differs from a harmonic signal. If \( \alpha = d\Omega/dt \) is small it may be assumed that the unbalance forces acting on the shaft are described by \( f_u = U\Omega^2\sin(\frac{1}{2}at^2) \), where \( U \) represents the product of eccentricity and unbalance mass. The envelopes
of the unbalance response of the sensor in the vertical direction in measurement plane D and the voltage applied to the vertical actuator in plane A over the current rotational frequency \( f(t) = \omega t / 2\pi \) is shown in Figure 4. Even though a high angular acceleration of \( a / \omega^2 \approx 4 \times 10^{-4} \) is applied, the performance is not altered significantly in comparison to the perfectly controlled system. For the sake of completeness, the unbalance response of the systems including the ordinary observer and the DO is also included in the figure.

![Figure 4: Unbalance response and control effort in time domain](image)

5 CONCLUSION

The main result of this article is that both the DO and the UIO are theoretically capable of estimating the system states of a rotating shaft excited by unbalances, while not altering the systems transfer behavior significantly. Nonetheless it can be stated, that the DO is the superior method. This is mainly because conditions for its implementation are less restrictive compared to those for UIOs, fewer sensors and a less accurate approximation of \( E \) are required.

A drawback of the DO is the inclusion of the rotational frequency in the observer matrix \( A' \). As a result, a time variant observer has to be implemented and the rotational frequency has to be available in the real time system. UIOs are linear time invariant and do not exploit the current rotational frequency of the shaft since arbitrary signals are allowed as disturbances.

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