Analysis of rotordynamic forces for high inlet pre-swirl rate labyrinth seals

R. G. Kirk, R. Gao
Department of Mechanical Engineering, Virginia Tech Blacksburg, USA

ABSTRACT
Labyrinth seals are widely used in various kinds of turbo machines and they can change the rotordynamic properties of the system by generating forces on the rotor. A linearized model has been widely used by previous researchers to describe the rotordynamic properties of the labyrinth seal. However recent predictions of the rotordynamic coefficients calculated with a CFD analysis approach has shown that the first order model could not accurately describe the characteristics for high pre-swirl cases; therefore a second order model including inertia terms was built and applied for rotordynamic analysis. The results confirmed the necessity of the second order model in high pre-swirl conditions.

1 INTRODUCTION
Labyrinth seals are widely used in turbo machines to reduce leakage flow. The stability of the rotor is influenced by the labyrinth seal as a result of the driving forces generated in the chambers of the seal. A linearized model [1-3] has been widely used to describe the rotordynamic properties of a labyrinth seal. Equation (1) shows the standard form used to model the assumed linear forces.

\[
\begin{bmatrix} F_x \\ F_y \end{bmatrix} = \begin{bmatrix} K & k \\ -k & K \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} C & c \\ -c & C \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}
\]

(1)

where \((F_x, F_y)\) are reaction forces working on the rotor; diagonal terms \((K, C)\) are direct stiffness and damping of the seal; off-diagonal terms \((k, c)\) are cross-coupled stiffness and damping of the seal; and the inertia terms are neglected. The influence of a labyrinth seal on the rotor can be predicted by using the four coefficients of stiffness and damping shown in Eq (1). For example, the increasing cross-coupled stiffness, \(k\), decreases the stability of the rotor; larger direct damping, \(C\), keeps the rotor stable while the other two coefficients, \(K\) and \(C\), have much less influence on stability [4].

Based on the first order model, previous researchers predicted rotordynamic coefficients of the labyrinth seal by three methods: experiments, bulk flow, and analysis using computational fluid dynamics (CFD). These former researchers have shown that the pre-swirl prior to the first tooth has been a significant influence on the rotordynamic properties of a labyrinth seal.

The experimental method has been an important approach to study labyrinth seals for many years. Bencher and Wachter [5] conducted experimental testing to determine rotordynamic coefficients for labyrinth seals and concluded that the tangential force in an eccentric labyrinth seal is caused by the circumferential flow.
Rajakumar and Sisto [6] tested the circumferential pressure distributions and forces in cases with and without pre-swirl and found that the pre-swirl has a significant influence on the forward whirling rotor for a short labyrinth. Childs and Scharrer [7] tested rotordynamic coefficients for both teeth-on-rotor and teeth-on-stator labyrinth seals and found that the stiffness and damping coefficients are very sensitive to pre-swirl. Recently, Wagner et al. [8] successfully tested rotordynamic coefficients of a compressor eye seal which has been very difficult to measure. The experimental results of forces vs. whirl speed clearly showed the inertial effects caused by the high pre-swirl.

The bulk flow method is actually a simplified finite volume approach. It was applied to model and solve the flow in the labyrinth seal by Iwatsubo [9] in 1980 for the first time. Wyssmann et al. [10] and Scharrer [11] developed the two-control-volume bulk flow model to predict the flow and rotordynamic coefficients in a labyrinth seal. Kirk [12] developed a special design computer program using the bulk flow method to predict the forces and the rotordynamic characteristics of a labyrinth seal, including the inlet and discharge leakage path influence on the entry swirl prior to the first tooth of the seal.

With the development of computer technology, computational fluid dynamics (CFD) has become an important method to predict the flow and forces in seals since the 1990s. Rhode et al. [13, 14] developed a three-dimensional finite difference approach and studied the influence of inlet swirl on the flow though a labyrinth seal. It was shown that the inlet swirl has a significant influence on the forces produced by the flow in seal. Hirano et al. [15] used both TASCflow and bulk flow codes to calculate forces and rotordynamic coefficients of a typical compressor eye seal and a steam turbine labyrinth seal. It was shown that the bulk flow method overestimates the rotordynamic forces in the labyrinth seal, compared to the TASCflow results.

Recent 3-D numerical prediction on the four coefficients conducted by Kirk and Gao [16] shows that the pre-swirl has very significant influence on rotordynamic properties of a labyrinth seal so that the traditional first order rotordynamic model cannot accurately describe the characteristics for high pre-swirl cases. Thus a second order model, including inertia terms, has been proposed for the rotordynamic analysis of labyrinth seals in the current research.

2 RECENT 3-D NUMERICAL PREDICTION

Compressor eye seals are usually short seals with usually four to seven teeth and very high pre-swirl. Their rotordynamic coefficients are more difficult to test via experiment due to their shortness. Recently Kirk and Gao [16] conducted a systematical CFD analysis on the flow in a typical five tooth eye seal as shown in Figure 1. In that paper, rotordynamic coefficients for tooth-on-stator configuration at different pre-swirl rates were documented. Since that publication, further calculations of the rotordynamic coefficients for tooth-on-rotor configuration have been completed. Rotordynamic coefficients for both tooth-on-stator and tooth-on-rotor cases will be discussed in the following sections of this paper.

In the numerical simulation, a series of eye seal models were built in Solid Works and meshed with ANSYS-ICEM. The simulations were conducted with the ANSYS-CFX employing $k$-$\varepsilon$ turbulence model and scalable wall function. A systematical error estimation approach was conducted. The numerical approach was justified by comparing with previous bulk flow results. All the detailed information are clearly documented in reference [16].
Figure 1. Typical labyrinth seal configuration in centrifugal compressors

All the rotordynamic analysis in the current paper is based on the results obtained by the numerical approach. Cases with different clearances and tooth configurations (tooth-on-stator and tooth-on-rotor) will be analyzed using the second order model.

3 THE FIRST AND SECOND ORDER MODELS

3.1 The first order model

In a typical rotordynamic analysis procedure for the labyrinth seal, the first order model given by eq (1) would be further simplified. Assuming that the rotor is whirling at the speed $\Omega$ with an offset of $\delta$ as shown in Figure 2, the location of the rotor can be given as

$$
\begin{align*}
x &= \delta \cos(\Omega t) \\
y &= \delta \sin(\Omega t)
\end{align*}
$$

(2)

For convenience, the frame can be defined in the way that the center of the rotor always locates on the $x$ axis at the initial condition as shown in Figure 2. Then by substituting eq (2) into eq (1), it can be simplified for initial condition ($t=0$) as

$$
\begin{align*}
F_r / \delta &= -K - c\Omega \\
F_t / \delta &= K - C\Omega
\end{align*}
$$

(3)

where $F_r$ is the seal force in the radial direction ($F_x$ actually) and $F_t$ is the seal force in the tangential direction ($F_y$ actually).

Equation (3) can be directly used for rotordynamic analysis by fitting data points of forces at different whirl speeds, as shown by solid lines in Figure 3 to Figure 6. Forces at different whirl speeds plotted in Figure 3 to Figure 6 are extracted from the fluid field results obtained by the numerical approach introduced in [16].

3.2 The second order model

The second order model including inertial terms can be given as

$$
\begin{bmatrix}
F_r \\
F_t
\end{bmatrix} =
\begin{bmatrix}
K & k & x \\
- k & K & y
\end{bmatrix}
+ \begin{bmatrix}
c & C & x \\
- c & C & y
\end{bmatrix}
+ \begin{bmatrix}
M & m & x \\
- m & M & y
\end{bmatrix}
$$

(4)
where \( M \) and \( m \) are respectively the equivalent direct and cross-coupled mass. They are generated by the flow and should be related to whirling speed, pre-swirl rate and etc. Applying the same approach used for simplifying eq (1), eq (4) becomes
\[
\begin{align*}
F_r/\delta &= -K - c\Omega^2 + M\Omega^2 \\
F_t/\delta &= k - C\Omega - m\Omega^2
\end{align*}
\]  

(5)

In Figure 3 and Figure 4, data points are specific radial and tangential forces at different whirl speeds for the tooth-on-stator case. Data points were approximated by the first order model with solid lines and also approximated by the second order model with dashed lines. The stiffness and damping can be obtained from those fitted curves. It is obviously shown that the first order model (solid lines) can fit the data points well when the pre-swirl rate is low (3.00%) but it gives very large error when the pre-swirl rate becomes high (73.6%). In this condition, the second order model (dashed lines) including inertial terms is required. Figure 5 and Figure 6 has the same curve approximation approach for the tooth-on-rotor case. The results agree with those of the tooth-on-stator case.

Figure 3 - Figure 6 show that for both tooth-on-stator and tooth-on-rotor configurations the first order model, eq (3), works well for low pre-swirl cases but leads to bigger and bigger error with the increase of pre-swirl rate; while dashed lines plotted based on the second order model can fit data points much better for high pre-swirl rates. Thus it is necessary to employ the second order model including inertia terms to describe the rotodynamic behaviors of the labyrinth seal for high pre-swirl cases. The two additional coefficients of equivalent direct and cross-coupled mass given by the second order model are usually not taken into account in a traditional rotodynamic analysis on a rotor-bearing-seal system. But the second order model can predict the stiffness and damping more precisely than the first order model and when implemented, it will improve the accuracy of the rotodynamic stability analysis on the system.

Figure 3. Predicted radial seal force vs. whirl speed at different pre-swirl rates (tooth-on-stator, 0.1 eccentricity)
Figure 4. Predicted tangential seal force vs. whirl speed at different pre-swirl rates (tooth-on-stator, 0.1 eccentricity)

\[ y = 1.8E+0x^2 - 6.3E+3x + 3.3E+6 \]
\[ y = -4.2E+3x + 3.0E+6 \]

Figure 5. Predicted radial seal force vs. whirl speed at different pre-swirl rates (tooth-on-rotor, 0.1 eccentricity)

\[ y = -5.8E+0x^2 + 9.7E+3x - 9.4E+6 \]
\[ y = 2.9E+3x - 8.4E+6 \]
Figure 6. Predicted tangential seal force vs. whirl speed at different pre-swirl rates (tooth-on-rotor, 0.1 eccentricity)

4 ROTORDYNAMIC ANALYSIS OF A TYPICAL COMPRESSOR EYE SEAL

The rotordynamic analysis on the typical compressor eye seal shown in Figure 1 was conducted employing the second order model for both tooth-on-stator and tooth-on-rotor configurations. Coefficients including inertia terms at different pre-swirl rates were predicted.

Figure 7 shows the influence of pre-swirl rate on the direct and cross-coupled stiffness. It can be concluded that the increasing pre-swirl leads to the increase of both direct and cross-coupled stiffness for both the tooth-on-rotor and tooth-on-stator configurations. The increasing cross-coupled stiffness is an unstable factor for the rotor of both tooth configurations. Moreover the tooth-on-rotor configuration has larger cross-coupled stiffness than the tooth-on-stator configuration, which indicates that the tooth-on-rotor model may be more unstable.

The influence of pre-swirl on the direct and cross-coupled damping can be seen in Figure 8 for both the tooth-on-rotor and tooth-on-stator configurations. The increasing pre-swirl leads to the increase of the direct damping but leads to the decrease of the cross-coupled damping. The increasing direct damping is a stable factor for the rotor of both tooth configurations. Furthermore the bigger cross-coupled stiffness and direct damping for the tooth-on-rotor model have works oppositely on the stability of the rotor and thus it is difficult to judge which tooth configuration is more stable.

Finally, Figure 9 shows the influence of pre-swirl on the direct and cross-coupled equivalent mass. It is shown that the increasing pre-swirl leads to the increase of both direct and cross-coupled equivalent mass for both the tooth-on-rotor and tooth-on-stator configurations. Note that the curves in Figure 9 are going down but the magnitude of equivalent mass is actually increased for increasing pre-swirl rate.
Figure 7. Predicted stiffness vs. pre-swirl rates

Figure 8. Predicted damping vs. pre-swirl rates
CONCLUSIONS

Considering the CFD and rotordynamic analysis results discussed in this paper, the conclusions of this work can be summarized as follows:

(1) The traditional first order rotordynamic model works well for low pre-swirl cases but does not accurately reflect the characteristics in high pre-swirl conditions. Thus a second order model including inertia terms is required to describe the rotordynamic behaviors of the labyrinth seal for high pre-swirl cases.

(2) For both the tooth-on-rotor and tooth-on-stator configurations, the direct and cross-coupled stiffness are increased with the increasing pre-swirl rate. The increasing cross-coupled stiffness will increase the potential for instability for both configurations. Moreover the tooth-on-rotor model has larger direct and cross-coupled stiffness, which could make the tooth-on-rotor configuration more unstable.

(3) For both the tooth-on-rotor and tooth-on-stator configurations, the direct damping is increased but the cross-coupled damping is decreased with the pre-swirl rate. The increase of direct damping serves to keep the rotor stable for both cases. Furthermore, the tooth-on-rotor model has larger direct damping than does the tooth-on-stator model, which could make the tooth-on-rotor configuration more stable. The function of larger cross-coupled stiffness and larger direct damping are opposite for the stability of the tooth-on-rotor model, so it would be difficult to state which tooth configuration is more stable, based on this information alone.

(4) For both the tooth-on-rotor and tooth-on-stator configurations, the magnitude of both direct and cross-coupled equivalent mass increase with the pre-swirl.
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REFERENCE LIST